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**ISSLEDOVANIE UPRUGIKH I PLASTICHESKIKH DEFORMATSII
MERZLYKH GRUNTOV**

**(AN INVESTIGATION OF ELASTIC AND PLASTIC
DEFORMATION OF FROZEN GROUND)**

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TABLE OF CONTENTS

	Page
Introduction	1
I. Types of Ground Used in Experiments	1
II. Method of Investigation	3
III. Resistance of Tested Frozen Ground to Outside Forces	6
IV. Basic Experiments on Elastic and Plastic Deformation of Frozen Ground	8
Elastic Deformation of Frozen Sand	8
Elastic Deformation of Frozen Silt	15
Elastic Deformation of Frozen Clay	18
V. Plastic Deformation of Frozen Ground and Its Temperature Border with Elastic Deformation	22
VI. Practical Deductions Based on Results of Studies	25

LISTS OF FIGURES

1. Compression curve for sand	2
2. Compression curve for silt	2
3. Compression curve for clay	3
4. Forms for preparation of samples	3
5. Setup of Aistov tensometers on a prism of frozen ground for measuring longitudinal deformation	4
6. Setup of Aistov tensometers for the simultaneous measurement of longitudinal and transverse deformations	5
7. Relationship between compressive strength of frozen ground and temperature	7
8. Relationship between shear strength of frozen ground under torsion and temperature	8
9-11. Relationship between Young's modulus for frozen sand and temperature	9,12
12. Relationship between the modulus of shear for frozen sand and temperature	14
13. Relationship between Young's modulus for frozen silt and temperature	15
14. Relationship between modulus of shear for frozen silt and temperature	18
15. Relationship between Young's modulus for frozen clay and temperature	19
16. Relationship between the rate of relative deformation of frozen ground and compressive stress	23

LIST OF TABLES

1. Mechanical composition of tested grounds	2
2. Typical moisture and specific gravity of tested grounds	2
3. Relationship between pressure and coefficient of porosity of tested grounds	2
4. Compressive strength of tested grounds	6
5. Shear strength of frozen grounds under torsion	7
6. Results of tests for the study of elastic and plastic deformations of frozen sands under compression	10,11
7. Elastic deformation and modulus of shear frozen sand under torsion	13
8. Poisson's ratio for frozen sand obtained by direct measurement	15
9. Elastic and plastic deformation of frozen silt under compression	16,17
10. Elastic deformation and modulus of shear of frozen silt under torsion	17
11. Results of the direct determination of Poisson's coefficient for frozen silt	18
12. Elastic and plastic deformation of frozen clay under compression	20,21
13. Elastic deformation and modulus of shear of frozen clay under torsion	22
14. Stabilized rate of relative deformation for frozen sand under compression	23
15. Stabilized rate of relative deformation for frozen clay under compression	24
16. Comparison of stabilized rate of relative deformation of frozen sand and clay under compression between -1.5° and -2.0°C	25

ISSLEDOVANIE UPRUGIKH I PLASTICHESKIKH DEFORMATSSI MERZLYKH GRUNTOV (An Investigation of Elastic and Plastic Deformation of Frozen Ground)

by

N. A. Tsytoich

INTRODUCTION

The deformation of frozen ground by external stresses is of major importance to the strength and stability of structures erected thereon. The loads to which a structure may be subjected fall into two categories: static and dynamic. Dynamic stresses have to be considered in designing machine foundations or in construction operations in areas subject to earthquakes, etc. In making computations for the foundations of structures which will be subject to the action of dynamic loads, a knowledge of the elastic properties of the foundation base, and in particular, of frozen ground, is absolutely essential.

The predominant type of static load is a constant load. Under given conditions, a constant load may cause sustained plastic deformation which will, in turn, result in gradual but uninterrupted structural strain, having a negative effect upon strength and stability.

The magnitude of both elastic and plastic deformation depends to a considerable degree upon the temperature of the frozen ground. There is, accordingly, a real need for research into the relation between the temperature and the elastic and plastic deformation of common types of frozen ground, and to determine the temperature at which a given deformation begins and ceases.

The question thus arises as to the temperatures at which elastic deformation predominates, and on the other hand, those which result in deformation of a primarily plastic nature.

This question must be taken, of course, in conjunction with the magnitude of the external stresses acting upon the frozen ground. Therefore, the studies reported on herein were made under the magnitudes of stress most frequently present in building foundations.

The experiments of the Permafrost Committee described below were conducted in such a manner as to maintain the required conditions of temperature both within and at the surface of the specimens. This has not been the case in previous investigations of deformation in frozen ground. This object was attained by conducting all the experiments in a special frost laboratory,¹ which housed both the specimens of frozen ground, the researcher himself, and the instruments, and within which a given temperature was maintained over an extended period of time.

Torsion, longitudinal, and transverse stresses were studied, and a number of special instruments were designed toward that end. Artificially frozen ground was used in all the experiments.

I. TYPES OF GROUND USED IN EXPERIMENTS

Clean sand, silt, and clay - clastic aggregates of the types most commonly encountered - were used in the experiments. Their clastic composition was as shown in Table 1. As shown by the data, the clean sand consisted primarily of particles 1 to 0.25 mm. in diameter; the silt contained 61.2% of silt-dust particles (0.05 to 0.005 mm.); and, finally, the clay consisted of 50% particles in the clay category (less than 0.005 mm. in diameter).

It is therefore clear that the samples selected were commonly encountered types of sharply differentiated clastic composition.

Table 2 shows typical rates of moisture content for the types of ground under study, namely, the Atterberg limits and initial maximum moisture content along the compression curve (at zero load). It further indicates true specific gravity of the particles of ground.

Given the data in Table 2, it is obvious that when the temperature was above freezing and the samples were therefore moist, the sand was not the least bit plastic, the silt was highly plastic and had a high

1. The experiments were conducted in the large frost chamber of the Leningrad Institute of Public Services Engineers (L.I.I.K.S.), which had been specially equipped for the experiments.

TABLE 1. MECHANICAL COMPOSITION OF TESTED GROUNDS.

Diameter of particles, mm.	Amount of particles, %		
	Sand	Silt	Clay
1-0.25	93.0	3.7	0.4
0.25-0.05	5.6	31.9	1.8
0.05-0.01	1.4	55.2	8.6
0.01-0.005		6.0	39.2
0.005-0.001		3.2	7.1
less than 0.001			42.9

TABLE 2. TYPICAL MOISTURE AND SPECIFIC GRAVITY OF TESTED GROUNDS.

Type of ground	Atterberg limits			Initial moisture under compression, %	True specific gravity
	Liquid Limit	Plastic Limit	Plasticity Index		
Sand	---	---	0	20.0	2.67
Silt	25.2	18.8	6.4	35.1	2.69
Clay	53.7	27.0	26.7	56.4	2.78

saturation point (35.1% initial moisture under compression), while the clay was very plastic with a high degree of saturation.

The relationship between pressure and porosity cited (or moisture content) - the so-called compression ratio - is a most important characteristic of ground at positive temperatures.

The data required for the determination of the compression relationship of the types of ground under study is set forth in Table 3 and given graphic expression in Figures 1, 2, and 3.

TABLE 3. RELATIONSHIP BETWEEN PRESSURE AND COEFFICIENT OF POROSITY OF TESTED GROUNDS.

Sand		Silt		Clay	
Pressure, P, kg./cm. ²	Coefficient of Void Ratio, e	Pressure, P, kg./cm. ²	Coefficient of Void Ratio, e	Pressure, P, kg./cm. ²	Coefficient of Void Ratio, e
0.00	0.700	0.00	0.944	0.00	1.568
0.50	0.688	0.52	0.683	0.45	1.156
1.00	0.681	0.88	0.621	0.84	1.109
2.00	0.672	1.66	0.602	2.50	0.953
5.00	0.650	5.19	0.551	4.73	0.876

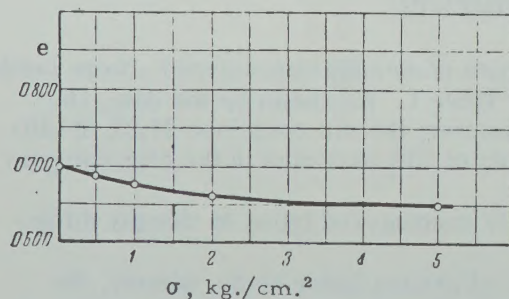


Fig. 1. Compression curve for sand.

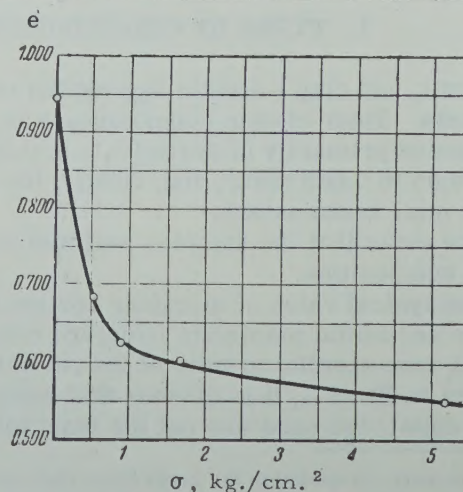


Fig. 2. Compression curve for silt.

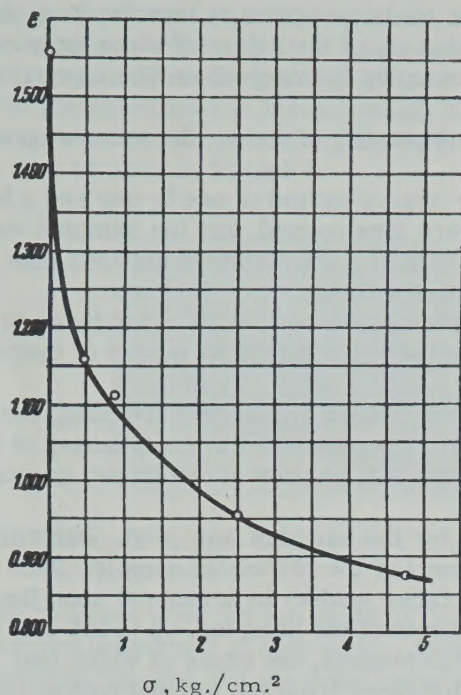


Fig. 3. Compression curve for clay.

described below we limited ourselves to a single degree of ice saturation, namely, that which corresponded approximately to the liquid limit when the ground was in a state of thaw. This is because many studies, including our own, have demonstrated that this degree of ice saturation, in which the pores in the ground are completely filled with ice and saturation approaches the point at which the ground will flow when thawed, is the degree most frequently encountered in the permafrost zone.

The setup of the experiments must be described in greater detail. In the first place, as stated above, artificially frozen ground was employed for all our tests.

Dry ground was sieved through a one-millimeter mesh screen, and then moistened to the required degree (54% for clay, 30% for silt, and 20% for sand). As the moisture content was equal to or greater than the point of fluidity, no packing was needed to get the desired type of sample.

The forms used to prepare the samples were of various shapes. Cubes measuring 5 cm. along each edge (Fig. 4a) were used for the compressive strength tests. Prisms measuring 7.1 x 7.1 x 15 cm. were used to study the deformation of frozen ground under compression (Fig. 4b). Cylindrical forms were used to measure deformations caused by torsion.

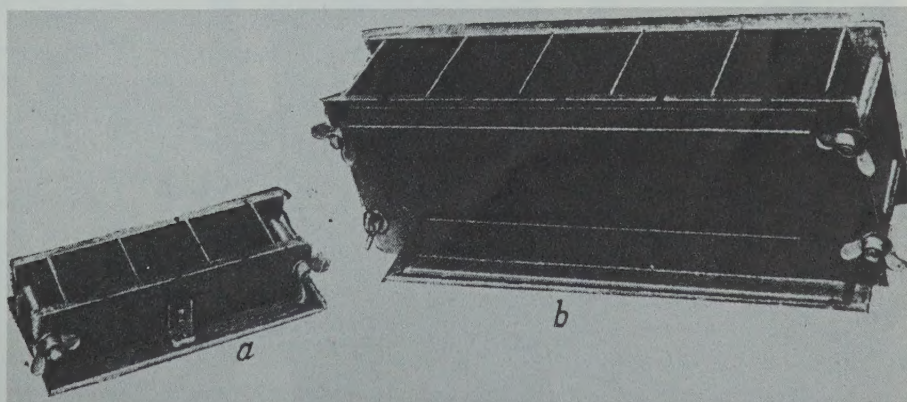


Fig. 4. Forms for preparation of samples.

A comparison of the compression curves shows that at positive temperatures sand has the least compressibility and clay the most. The compressibility of silt is midway between that of clay and sand.

It must also be noted that the quantity of water needed to fill the pores in the ground varies with the total volume of those pores, which is conditioned, in turn, by the load borne by the ground.

II. METHOD OF INVESTIGATION

As had already been established by previous studies of elastic and plastic deformation of frozen ground, there are several factors contributing to the degree of deformation caused by compression.¹ The most important of these factors are temperature, ice content, and the conditions under which the experiments are conducted (shape of samples, process of application of pressure, etc.).

The factor given major consideration in our work was temperature. Therefore, studies of the deformation of samples of frozen ground were made at -0.5° , -1.5° , -5° , and -10°C . This range covers almost completely the temperatures of permafrost encountered in nature.

Ice saturation is another matter entirely, for here the possible variations are exceedingly great. In the experiments

1. N. A. Tsytoich and I. S. Vologdina, "Opredelenie uprugikh postoiannykh merzlykh gruntov i issledovanie ikh svoistv plastichnosti" (Determination of Elastic Constants of Frozen Ground and Investigation of Their Plastic Properties), Sb. II Labor. issled. mekhanich. svoistv merzlykh gruntov (Laboratory Researches into the Mechanical Properties of Frozen Ground), Vol. II, Academy of Sciences, Moscow and Leningrad, 1936.

The torsion tests of frozen ground were supplementary to the initial program of investigation, inasmuch as only such tests can provide a measure of pure shear; that is, of the action of shearing stresses alone, if we ignore the negligible influence of the weight of the samples themselves on the unequal distribution of the stresses within them.

To prepare the samples, the soil was mixed with the required quantity of water; the mix was poured into the forms, and it was frozen within them.

The freezing in these forms took place in the small chamber over a period of one to one and a half days, at temperatures ranging from -25° to -5°C . The forms were then opened, and the samples were removed and transferred to a larger chamber where they were kept at a temperature not less than that of the test and not higher than -0.5°C ., for about two days prior to the tests.

On the day of the tests, the samples were kept at the desired temperature ($\pm 0.2^{\circ}\text{C}$.) for three to four hours, during which time the temperature of the sample was read at least hourly by means of thermocouples.

Due to the common phenomenon of redistribution of moisture in ground undergoing freezing, the samples were not, it must be pointed out, entirely identical. This later resulted in a scattering of the points on the curves, which expressed the relationship between the deformation of the frozen ground and the degree of negative temperature and other factors.

In determining compressive strength, a standard cubic form for the samples was used. Deformation of frozen ground was measured only on the central section (10 cm.) of the prismatic sample. This was done to escape the adverse influence of friction (along those surfaces subject to pressure) upon the uniformity of the deformation to be studied. The upper and lower parts of the samples, up to 2.5 cm. in length, were excluded from consideration. For this purpose, iron plaques, the edges of which had threaded openings, were placed inside the samples at specific distances from the top and bottom by means of a special tool. After the samples were frozen, metal buttons with copper heads were screwed into these openings. Tensometers or mirror apparatus were attached to these buttons to measure the deformation.

Therefore, only the middle portion of the samples of frozen ground, remaining between these plaques, was used for the tests. The length of the investigated portion of the prism in all experiments was equal to 10 cm.

The calibration of all the recording instruments - such as Professor Aistov's tensometers for the main experiments and Martens' mirror apparatus for the control experiments - was done in the large chamber, at negative temperatures. The tests were conducted primarily by means of hand-lever presses (with a capacity of 0.5 to 3 tons). Some use was made of a 5-ton Amsler oil press.

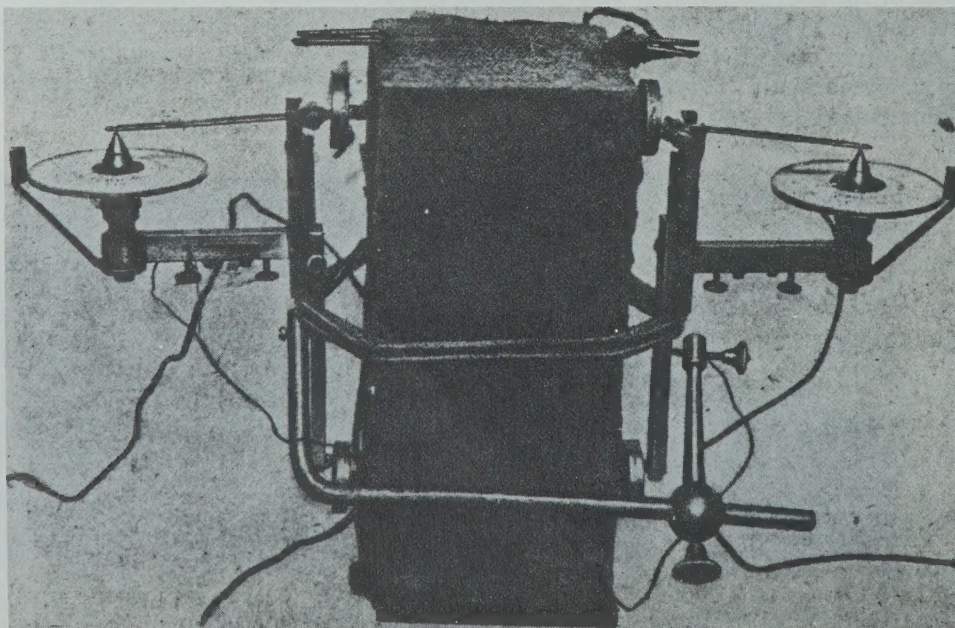


Fig. 5. Setup of Aistov tensometers on a prism of frozen ground for measuring longitudinal deformation.

Figure 5 shows Aistov tensometers in use on a prism of frozen ground. This photograph was taken in the laboratory at a positive temperature. This caused a slight thawing of the surface of the sample, which is apparent in the photograph. No such thawing occurred during the experiments.

For the simultaneous measurement of longitudinal and transverse deformation of the prisms of frozen ground by compression, M. P. Vinogradov designed a special base for the Aistov tensometers. This may be seen in Figure 6.

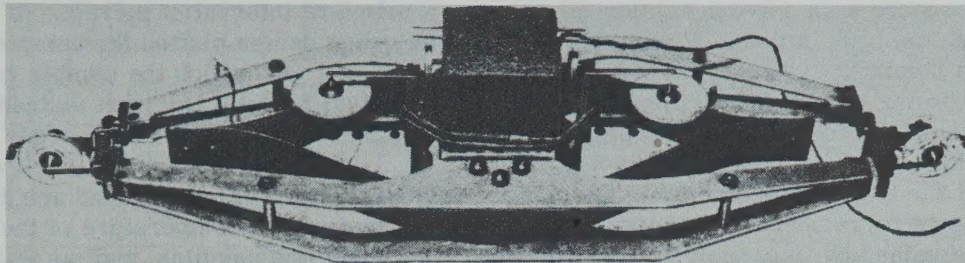


Fig. 6. Setup of Aistov tensometers for the simultaneous measurement of longitudinal and transverse deformations.

The samples used for the study of deformation of frozen ground by torsion were cylindrical in the middle and broadened at both ends. Also, the middle of each sample was free from the metal form during the experiments, but the ends remained in the iron container. Only the deformation of the middle of the sample 5 cm. in length and 4 cm. in diameter was studied. This portion was 2.5 cm. from the wider part of the samples and 7.5 cm. from the end.

Studies of deformation were conducted under several different loads. The load interval was taken at $1/10$ to $1/20$ of the compressive strength of the sample to the given stress. At that, the four intervals comprised only between 0.4 to 0.2 of the compressive or shear strengths of the sample at the standard rate of load increase. This choice of loads permitted the testing of almost every sample at not less than four different loads. At that, a complete mechanical fluidity of the frozen ground was often observed even at loads considerably less than half the compressive strength of the sample of frozen ground.

The circumstances outlined above made it necessary to determine the compressive and torsion strengths (pure shear) of each batch of samples separately.

No less than three samples of each type of ground were tested at a given temperature, while the degrees of elastic deformation cited below were obtained as the average of twenty tests. The usual procedure for the tests was the following. First, compressive strength was established by testing at least three samples, and the load intervals were chosen. The intervals ranged up to 0.5 to 1 kg./cm.² and in some cases several kilograms for a given sample. Then, elastic deformation of two prisms was studied simultaneously, and the reading for each load was repeated not less than five times, both as load was increased and decreased.

After a series of cycles of loading and unloading, the deformation of each prism was studied. Not less than one hour was permitted for each load to take effect.

The next day a control experiment would be set up. One or two more prisms were tested at the same temperature and at the same load intervals. One of the prisms was studied both for longitudinal as well as for transverse deformation by compression.

During each test, the temperature of the sample of frozen ground was read regularly by means of thermocouples, approximately every half hour and during every change in method of loading or testing.

If the resulting temperature was considerably different from that desired (about $0.5^{\circ}\text{C}.$), the test was stopped until regulation of the temperature in the chamber caused the sample to reach the desired temperature (within $\pm 0.2^{\circ}\text{C}.$). Deformation caused by torsion was also studied under at least three different cycles of loading and unloading and for a considerable period under each load.

Besides the above-mentioned tests, a number of experiments were made to study deformation caused by pressing a hard stamp into the sample of frozen ground.

To fulfill the outlined program, a considerable number of measurements of deformation of frozen ground were made (not less than 15,000 separate measurements), which required much work and time.

III. RESISTANCE OF TESTED FROZEN GROUND TO OUTSIDE FORCES

As was pointed out above, samples of ground used for compression tests were frozen at a temperature of about -25°C . and were then kept at the desired temperature for several hours prior to the test and during the entire time of the test. Also, the method of freezing was the same as for the prisms of frozen ground, the deformation of which was studied in detail.

Cubes of frozen ground measuring 5 cm. along each edge were used for tests of compressive strength.

The tests were made with a 5-ton Amsler oil press at a standard interval of increase of compressive load of 15 to 20 kg./cm.² per minute. During the tests, a recording device plotted the compression diagram. Maximum compressive strength was taken to be the largest load which the sample could withstand without becoming plastic or attaining the limit of mechanical fluidity; i.e., that amount of stress under which deformation constantly increased without further increase in load. The second limit was determined graphically on the compression diagram.

The results of the tests for determining the compressive strength of frozen ground are given in Table 4, where each figure represents the average of at least three tests. The temperature of the ground samples was determined by means of thermocouples placed inside the samples, and, as was pointed out above, the temperature of the surrounding air was close to that of the temperature of the sample.

TABLE 4. COMPRESSIVE STRENGTH (MECHANICAL LIMIT OF FLUIDITY) OF TESTED GROUNDS AT THE RATE OF INCREASE OF COMPRESSIVE LOAD $v=15$ to 20 kg./cm.²/min.

Date of testing	Moisture by weight, w, %	Temperature of sample, $^{\circ}\text{C}$.	Compressive strength, σ_b , kg./cm. ²
Sand			
5/7/37	16.8	— 9.0	127
5/3/37	17.2	— 3.4	67
4/25/37	17.0	— 2.9	64
12/17/37	18.1	— 0.5	9
Silt			
5/13/37	27.1	—10.3	128
5/3/37	28.2	— 5.1	78
11/16/37	30.0	— 1.8	36
11/25/37	27.6	— 0.5	9
Clay			
5/16/37	54.6	— 8.2	45
4/25/37	53.7	— 3.4	23
12/1/37	54.0	— 1.5	13
12/7/37	52.8	— 0.5	9

The data provided in Table 4 were used to erect a graph of the relation between compressive strength and temperature (Fig. 7).

From these data, it may be seen that frozen sand has the greatest resistance to pressure; frozen silt has a compressive strength close to that of sand; while frozen clay has a much smaller resistance than does sand (2 to $2\frac{1}{2}$ times less).

The relation of σ_b to the value of the negative temperature for frozen sand and silt is expressed by a curve. As far as artificially frozen clay is concerned, the relationship between the resistance to pressure at the liquid limit and temperature is linear within the limits covered by our study.

Let us also point out an extremely interesting, practical, and very important fact. Despite the variations in grain-size composition and ice content, all the samples of artificially frozen ground had the same compressive strength (mechanical limit of fluidity) at a temperature of -0.5°C .

Therefore, at temperatures close to 0°C . (-0.5°C .), the grain-size composition of frozen ground whose pores are filled with ice has almost no influence on its compressive strength. The influence of

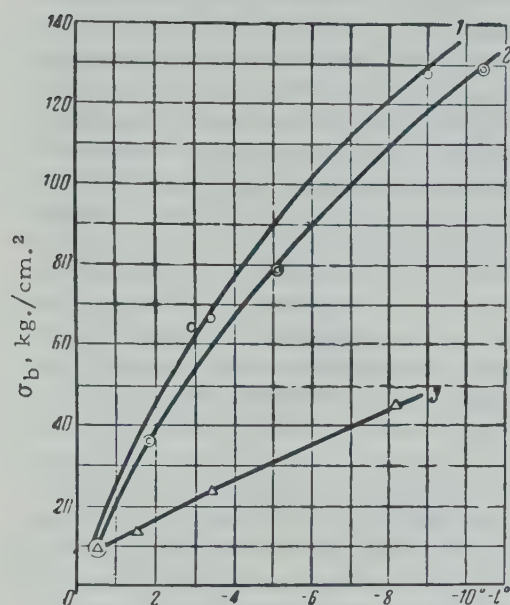


Fig. 7. Relationship between compressive strength of frozen ground and temperature. (Curve 1, frozen sand; curve 2, frozen silt; curve 3, frozen clay.)

grain-size composition becomes sharper with reduced temperatures. An especially sharp difference is observed between frozen sand and frozen clay.

Inasmuch as any force acting on a certain area may be considered the resultant of perpendicular and tangential forces prime importance attaches not only to compressive but also to shear strength. The latter may be studied by testing the samples of frozen ground to destruction by torsion.

Table 5 gives the results of experiments (apparently never previously undertaken) on the resistance of frozen ground to pure shear by torsion. The majority of the tests were made with a rate of load increase equal to about 1.4 to 1.5 kg./cm.² per minute. However, several samples were tested at a ten-fold greater rate of increase, i.e., with $v \approx 15$ kg./cm.² per minute.

The results of these experiments, given in Table 5, and presented graphically in Figure 8, shows the relationship between temperature and shear strength of frozen ground under torsion.

The data obtained (Table 5 and Figure 8) on the shear strength of frozen ground under torsion show that the maximum stress that can be withstood before destruction depends to a great degree on the temperature of the sample, increasing in inverse ratio to temperature on a curved line (Fig. 8).

The curves which express the relationship between shear strength and temperature have the same shape for all types of ground investigated. Only the parameters of the curves change.

TABLE 5. SHEAR STRENGTH OF FROZEN GROUNDS UNDER TORSION.

Moisture by weight, w, %	Temperature of sample, °C.	Rate of load increase, v, kg./cm. ² /min.	Shear strength, kg./cm. ²
Frozen sand			
16.4	—11.0	1.4—1.5	47.6
16.3	—10.9		47.7
17.1	— 2.5		17.1
17.1	— 2.6	15.0	24.4
18.4	— 1.5	1.4—1.5	11.4
18.4	— 1.5		10.8
16.8	— 1.2		9.1
20.0	— 0.5		4.8
Frozen silt			
26.4	— 9.5	1.4—1.5	39.7
29.8	— 6.0		27.7
28.6	— 2.9		16.5
28.6	— 1.8	1.4—1.5	9.7
28.6	— 1.3		6.8
26.2	— 0.5		4.8
Frozen clay			
51.9	—10.0	1.4—1.5	18.1
53.7	— 5.0		11.4
52.8	— 1.9		6.8
54.1	— 1.4	1.4—1.5	6.5
55.4	— 0.5		4.6

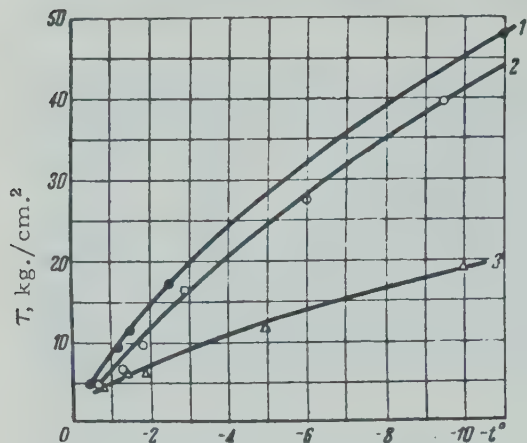


Fig. 8. Relationship between shear strength of frozen ground under torsion and temperature. (Curve 1, frozen sand; curve 2, frozen silt; curve 3, frozen clay.)

As experiments have shown, the shear strength at which destruction occurs, as determined by the tests, depends on the rate of load increase. Thus, the resistance to shear by torsion for frozen sand, saturated with ice at a rate of load increase of 1.4 to 1.5 kg./cm.² per minute and a temperature of -2.5°C., was 17.1 kg./cm.²; at a rate of load increase of 15 kg./cm.² per minute and at the same temperature, shear strength rose to 24.4 kg./cm.²; i.e., an approximate tenfold increase in rate of load resulted in a shear strength about 1 1/2 times as great.

Let us note that, according to Figure 8, the lower the temperature of the sample, the more sharply the influence of grain-size composition is manifested on shearing strength. However, at a temperature of about -0.5°C., all types of ground subjected to study showed virtually the same shearing strength.

Therefore, our study of the shearing strength of frozen ground reveals a phenomenon identical to that observed during the tests of compressive strength. At temperatures close to zero, all the types of ground investigated show practically the same resistance to the action of outside forces, regardless of grain-size composition.

IV. BASIC EXPERIMENTS ON ELASTIC AND PLASTIC DEFORMATION OF FROZEN GROUND

Elastic and plastic deformation of three types of frozen ground, typical in their composition, were studied by the application and removal of weight to prisms of frozen ground.

The degree of elastic deformation was determined at various load intervals. The process of deformation in relation to time was likewise studied under different compressive loads. The elastic deformation of any homogeneous body is entirely determined by its elastic constants - Young's modulus and Poisson's ratio. These factors are taken as our point of departure in studying elastic deformation.

The "stabilized rate of relative deformation" is taken as the index of plastic deformation of frozen ground under compression. The equation for the stabilized rate of relative deformation is:

$$\bar{m} = \frac{\lambda}{u}$$

where \bar{m} is the stabilized rate of deformation and λ is the relative deformation, which is equal to the ratio of full deformation of the prism to its initial length, i.e.,

$$\lambda = \frac{\Delta \ell}{\ell}$$

where $\Delta \ell$ is full deformation, ℓ is the length, and u is the time required for deformation λ to occur.

The stabilized rate of deformation was understood as that at which the deformation per unit of time remained practically constant for at least an hour.

Elastic Deformation of Frozen Sand

The results of tests on elastic and plastic deformation of frozen sand under compression are given in Table 6.

Effect of Variations in External Stress

In studying the data of Table 6, attention must be paid first to the relationship between the elastic deformation of frozen sand (or more correctly, its Young's modulus) and the compressive stress. Thus, the tests in Series I show that at a temperature of -10°C. Young's modulus decreased from 200,000 to 190,000 kg./cm.² These tests were made with the lever press.

A similar phenomenon was also observed during tests on prisms of frozen sand using the oil press. Thus, in tests 9 to 12, application of the full load within 10 seconds and with a change of compressive stress from 10 to 40 kg./cm.², the modulus of elasticity of the frozen sand changed comparatively little - from 217,000 to 181,000 kg./cm.² In these tests, the rate of load application was increased with increasing compressive stress. However, when the rate of load increase does not change, as, for example, in tests 13 to 16, the change in Young's modulus was considerably more intense. With an increase in compressive stress from 10 to 40 kg./cm.², Young's modulus for frozen sand decreased from 200,000 to 82,000 kg./cm.²; i.e., to less than half. Therefore, under the action of considerably larger compressive forces, the rigidity of frozen sand decreases, and the deformation under the influence of the same impact of the outside forces increases.

The relationship between Young's modulus for frozen sand and the compressive stress becomes still more striking as the temperature of the frozen ground rises. Thus, according to tests 27, 28, and 29, at a temperature of about -2.2°C., the modulus of elasticity changed from 71,400 to 45,000 kg./cm.² with a change of compressive stress from 1 to 3 kg./cm.² Likewise, according to tests 51, 52, and 53, with increase in compressive stress from 1 to 3 kg./cm.², at a temperature varying from -0.6° to -0.4°C., Young's modulus decreased from 13,200 to 8,000 kg./cm.²

Therefore, when the temperature of frozen sand samples is higher and approaches zero, the influence of compressive stress on elastic deformation (the values of Young's modulus) is more strongly manifested than at lower temperatures, ranging from -5° to -10°C.

Let us also point out that in frozen sand, the relationship between Young's modulus and compressive stress is especially well manifested when the stress is small (ranging from 1 to 5 kg./cm.²). At greater pressure, however, it appears as if a certain consolidation takes place, and the rate of change in Young's modulus with change in compressive stress is not so considerable.

The latter fact is also confirmed by tests 46 to 50, which were performed with the Amsler oil press.

Relationship Between Young's Modulus and Temperature of Frozen Sand

In a study of the relation between elastic deformation in frozen sand and changes in its temperature, the degree of external stress must be given due consideration. Consequently, on the basis of data in Table 6, the relationship between temperature and Young's modulus for frozen sand at a stress of 2 kg./cm.² is presented graphically in Figure 9.

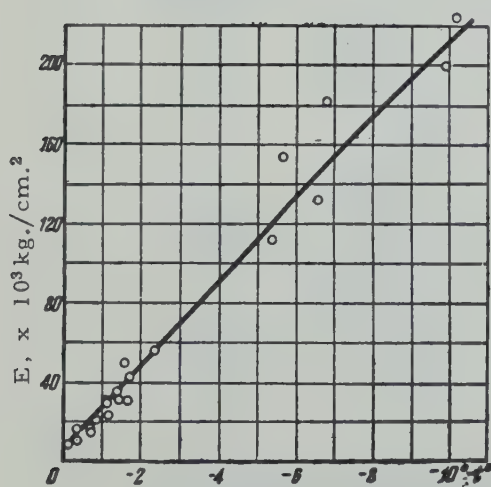


Fig. 9. Relationship between Young's modulus for frozen sand and temperature (down to -10°C.) with stress of $\sigma = 2$ kg./cm.²

Figure 9 shows that the relationship between temperature and Young's modulus for frozen sand is linear within certain specific limits, a fact which we have already noted.¹

The lower limit of the negative temperature will be about -5° to -7°C. For the frozen sand under study, with pores completely filled with ice, this linear relationship holds down to -5° or -7°C. However, for the entire range of temperatures studied, the relationship of the modulus of elasticity of frozen sand to temperature is expressed by a more complex law:

$$E = \alpha + \beta t + \gamma t^2 + \delta t^3$$

where α , β , γ , and δ are constants, and t is the exact negative temperature in °C.

An examination of the shear modulus and Poisson's ratio of sand convinces us that the relationship between Young's modulus and negative temperature is curvilinear when the temperature fluctuation is great. However, according to Figure 9, both curvilinear and linear relationships have the same degree of probability.

Within the limits of the temperature range from -0.2° to -5.0°C., the coefficients for the terms with powers greater than 1.0 may be taken as equal to zero, for all practical purposes.

1. N. A. Tsytoich and M. I. Sumgin, "Osnovaniia mekhaniki merzlykh gruntov" (Principles of Mechanics of Frozen Ground), Academy of Sciences, 1937.

TABLE 6. RESULTS OF TESTS FOR THE STUDY OF ELASTIC AND PLASTIC DEFORMATION OF FROZEN SANDS UNDER COMPRESSION

Test No.	Series	Moisture by weight, -w, %	Temperature of sample, °C	Time of load action, u	Stress σ kg./cm. ²	Young's modulus, E, kg./cm. ²	Stabilized rate of relative deformation, m, min. ⁻¹
1	2	3	4	5	6	7	8
1 ¹	I	13.8	-10.2	30" (5 times)	2	225 000	0
2		13.8	-10.2		4	224 000	0
3		15.7	-9.9		2	200 000	0
4		15.7	-9.9		4	194 000	0
5		15.7	-9.9		6	193 500	0
6		15.7	-10.6		10	190 100	0
7		15.7	-10.8	after 60'	10	190 000	0
8		15.7	-9.7	after 2760'	10	161 300	0
9 ²	I	16.0	-9.5	10" (5 times)	10	217 000	—
10		16.0	-9.5		20	200 000	—
11		16.0	-9.5		30	200 000	—
12		16.0	-9.5		40	181 000	—
13		16.0	-9.4	250" (5 times)	10	200 000	—
14		16.0	-9.4	500" (5 times)	20	142 000	—
15		16.0	-9.4	750" (5 times)	30	110 000	—
16 ³		16.0	-9.4	1000" (5 times)	40	82 000	—
17	II	17.1	-6.8	30" (5 times)	2	182 800	0
18		17.1	-6.8		3	182 000	0
19		17.1	-6.4		4	180 000	0
20 ⁴		17.1	-5.1		5	—	0.0000012
21		16.9	-5.0	60'	5	—	0.0000014
22		16.9	-5.1		6	—	0.00000225
23		15.9	-6.6	30" (5 times)	2	133 000	0
24		15.9	-7.2		4	129 000	0
25	III	17.3	-5.7		2	154 000	—
26		17.3	-5.5		3	176 500	—
27		21.9	-2.3	30" (5 times)	1	71 400	—
28		21.9	-2.2		2	57 100	—
29 ⁶		21.9	-2.0		3	45 000	0.000024
30 ⁷		21.9	-1.8	150'	4	—	0.000045
31		21.8	-1.2	30" (5 times)	1	27 000	—
32		21.8	-1.2		2	24 800	—
33 ⁸		21.8	-1.8		3	29 400	0.000019
34		21.8	-1.4	60'	4	—	0.000044
35 ⁹		21.8	-1.7	70'	2	—	0.000004

1. Tests 1 to 8 were done on the lever press. Complete cessation of deformation was observed ($v=0$).
2. Tests 9 to 16 were done on the Amsler oil press; tests 9 to 12 during a rapid increase of load; tests 13 to 16 with a constant rate of load increase, $v=2.4$ kg./cm.²/min.
3. No fluidity was observed even at $\sigma=60$ kg./cm.²
4. In test 20, plastic flow was observed at a constant load. In tests 20 and 21, the deformations were within the margin of error of the readings.
5. More accurate data
6. Measurements during 60'.
7. Average of 56 measurements.
8. Measurement every 5' for 60'.
9. In tests 35 to 37, with a constant load, measurement of deformation took place every 5' for 60'.

TABLE 6, continued.

Test No.	Series	Moisture by weight, w, %	Temperature of sample, °C	Time of load action, u	Stress σ kg./cm. ²	Young's modulus, E, kg./cm. ²	Stabilized rate of rela- tive deforma- tion, m, min. ⁻¹
1	2	3	4	5	6	7	8
36		21.9	-1.5	30" (5 times)	2	29 800	0.000006
37		21.9	-1.8		3	35 700	0.000019
38		21.9	-1.3		4	—	0.000041
39 ¹		19.4	(-1.8)		1	46 700	—
40		19.4	(-1.8)		2	42 000	—
41		19.4	(-1.2)		3	36 000	0.000034
42		19.4	-1.6		2	50 000	—
43		19.4	-1.2		2	23 000	—
44		19.4	-1.5		2	49 000	—
45		19.4	-1.2		2	31 300	—
46 ²	III'	17.8	-3.8	10" (5 times)	1.3	60 200	—
47		17.8	-3.8		5.5	43 200	—
48		17.8	-3.8		7.5	41 000	—
49		17.8	-3.8		9.8	36 000	—
50		17.8	-3.8		14.0	31 400	—
51	IV	21.8	-0.6	30" (5 times)	1	13 200	—
52		21.8	-0.4		2	10 600	0.000051
53		21.8	-0.4		3	8 000	—
54 ³		21.8	-0.4	60'	1	—	0.000011
55		21.8	-0.6		3	—	0.000097
56		21.9	-0.8	30" (5 times)	2	14 600	—
57		21.9	-0.6		3	10 800	0.000092
58		21.9	-0.8	60'	1	—	0.000009
59		21.9	-0.8		2	16 000	0.000038
60		19.2	-5.4	30" (5 times)	2	133 000	—
61		19.2	-1.7		2	37 800	—
62		19.2	-0.2		2	8 200	—
63		19.4	-0.4	30" (5 times)	1	18 900	—
64		19.4	-0.4		2	15 800	—
65		21.9	-0.9		1	20 400	—
66		21.9	-0.9	30" (5 times)	2	20 100	—
67 ⁴		21.9	-0.7		3	19 500	0.000050
68 ⁵	V	17.8	-1.5	60'	1	—	0.000000
69		17.8	-1.8		1.5	—	0.000002
70		17.8	-1.8		2	—	0.000008
71		17.8	-1.5		2.5	—	0.000013
72		19.4	-1.5		3.5	—	0.000027
73		19.4	-1.5		5	—	Complete loss of stability
74	Control	19.0	-1.7	30" (5 times)	2	29 400	
75		19.0	-1.5		3	24 600	
76		18.9	-1.7		0.5	41 700	
77 ⁶		18.9	-1.4		1	39 000	
78		18.9	-1.4		2	35 400	
79		18.9	-1.2		2	32 300	

1. In tests 39 to 41, the ground temperature in parentheses corresponds to the temperature of the chamber.

2. Series III' was done on the Amsler oil press with a rapid increase of load.

3. At $u=60'$.

4. At $u=60'$.

5. For tests 68 to 73, the average of two measurements is given.

6. Measurement of deformation in tests 77 and 78 was done with Martens mirror apparatus.

In this case, the equation for Young's modulus at a temperature not lower than -5°C . may be considered linear, i.e.,

$$E = \alpha + \beta t$$

To determine the parameters of a straight line, α and β , the initial portion of the curved line of Figure 9 is shown on a larger scale in Figure 10.

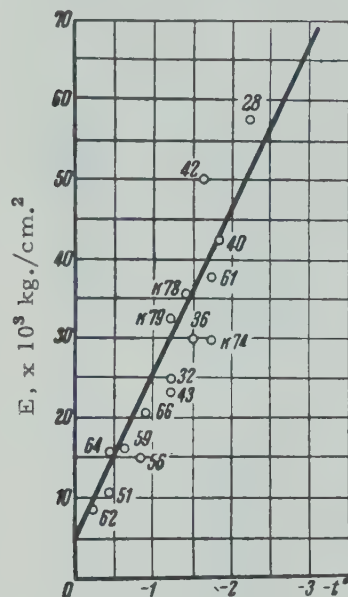


Fig. 10. Relationship between Young's modulus for frozen sand and temperature (down to -3°C .) with stress of $\sigma = 2 \text{ kg./cm.}^2$ ($E \approx (0.5 + 2.1t) \times 10^4 \text{ kg./cm.}^2$)

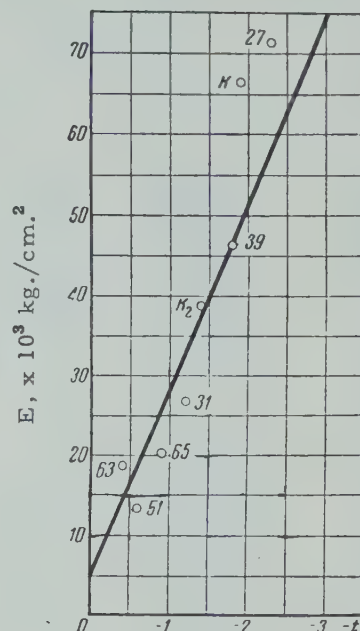


Fig. 11. Relationship between Young's modulus for frozen sand and temperature (down to -3°C .) with stress of $\sigma = 1 \text{ kg./cm.}^2$ ($E \approx (0.5 + 2.3t) \times 10^4 \text{ kg./cm.}^2$)

Determining graphically the parameter of line $E = f(t)$ of Figure 10, we have the following equation of the frozen sand studied under a compressive stress of 2 kg./cm.^2 :

$$E \approx (0.5 + 2.1t) \times 10^4 \text{ kg./cm.}^2$$

where t is the exact negative temperature in $^{\circ}\text{C}$.

Figure 11 shows the relationship $E = f(t)$ for the same frozen sand with approximately the same degree of ice saturation as in the tests, the results of which are shown in Figures 9 and 10, but with the value of compressive stress σ equal to 1 kg./cm.^2

For the case under consideration, according to the data of Figure 11, we would have:

$$E \approx (0.5 + 2.3t) \times 10^4 \text{ kg./cm.}^2$$

Comparing this equation with the previous one, we see that it differs only by its angle coefficient. Specifically, the modulus of elasticity of frozen sand under an external stress of 2 kg./cm.^2 will have an angle coefficient for line $E = f(t)$ of $\beta = 2.1$.

However, the angle coefficient for Young's modulus determined at various temperatures but with an outside pressure of $\sigma = 1 \text{ kg./cm.}^2$ is $\beta = 2.3$.

A similar calculation of the parameter of line $E = f(t)$ but with an outside pressure σ equal to 0.5 kg./cm.^2 resulted in $\beta \approx 3.3$, while α had its former value.

Therefore, the above data clearly indicate that the influence of temperature on elastic deformation of frozen sand increases inversely with outside pressure; i.e., the intensity of increase in Young's modulus with reduced temperature will be greater, the smaller the outside pressure.

Modulus of Shear and Poisson's Ratio for Frozen Sand

In addition to elastic deformation by compression, elastic deformation due to torsion was also studied. Table 7 gives the results of the tests for elastic deformation under torsion, and the value of the elastic increase in torsion angle, $\Delta\psi$, is given as an average of at least five tests.

The modulus of elastic shear was determined on the basis of the data for the elastic angle of torsion, the values of load, the length of the arm of the load application, and the diameter of the sample, according to the formula:

$$G = \frac{Pa\ell}{\Delta\psi I_{\rho}}$$

where P is the external load in kilograms, a is the length of the arm of load application (16.6 cm.), ℓ is the length of the portion of the rod subjected to test (5.0 cm.), $\Delta\psi$ is an elastic increase in torsion angle with change of load from P to zero, and I_{ρ} is the polar moment of inertia of the cross-sectional area of the rod (about 29.3 to 30.6 cm.⁴).

The values for modulus of shear obtained by the above method are given in Table 7.

TABLE 7. ELASTIC DEFORMATION AND MODULUS OF SHEAR FOR FROZEN SAND UNDER TORSION

Test No.	Moisture by weight, w, %	Temperature of sample, °C.	Load on sample, P, kg.	Greatest shear stress, τ , kg./cm. ²	Elastic increase of torsion angle, $\Delta\psi$ where $\ell = 5$ cm.	Modulus of shear, G, kg./cm. ²
1	16.9	—10.6	5	5.7	0.00012	113 400
2	16.9	—10.6	10	11.4	0.00031	88 000
3	16.9	—10.6	15	17.1	0.00062	66 000
4	16.9	—10.6	20	22.8	0.00112	48 500
5	18.1	—6.0	2	2.3	0.00011	51 400
6	18.1	—6.0	4	3.4	0.00024	47 100
7	18.1	—6.0	6	6.8	0.00040	42 300
8	18.1	—6.0	8	9.1	0.00059	38 300
9	17.2	—4.2	4	3.4	0.00026	41 800
10	17.2	—4.2	6	6.8	0.00050	32 600
11	17.2	—4.5	8	9.1	0.00071	30 600
12	17.2	—4.5	10	11.4	0.00086	31 600
13	18.1	—3.6	2	2.3	0.00018	30 200
14	18.1	—5.0	8	9.1	0.00076	28 600
15	17.9	—1.0	2	2.3	0.00050	10 900
16	17.9	—1.2	3	3.4	0.00064	12 700
17	17.7	—1.5	1	1.1	0.00016	17 000
18	17.7	—1.5	2	2.3	0.00034	16 000
19	18.0	—0.5	1	1.1	0.00055	4 000
20	18.0	—0.5	2	2.3	0.00136	4 000

From a study of the data in Table 7, we come to the conclusion that the modulus of shear of frozen sand depends on the same factors as does Young's modulus, previously discussed. Specifically, the modulus of shear decreases with increase of outside pressure and consequently with increase of shear stresses caused by outside load. The modulus increases, however, with decrease of temperature.

To study the influence of temperature the modulus of transverse elasticity (modulus of shear), we need only compare data obtained under the same loads, i.e., in a given case, under the same shear stress.

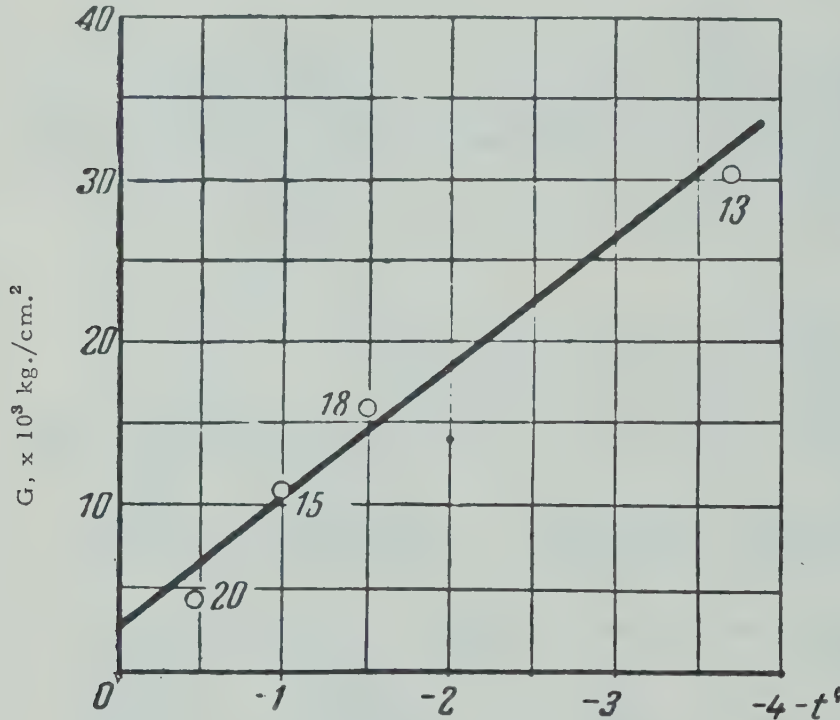


Fig. 12. Relationship between the modulus of shear for frozen sand and temperature when $\tau = 2.3$ kg./cm.² [$G \approx (0.2 + 0.8t) \times 10^4$ kg./cm.²]

ly, it can be concluded that Poisson's ratio depends on the same factors as do Young's modulus and the modulus of shear.

From the last formula, we have:

$$\mu = \frac{E}{2G} - 1$$

This last equation may also be utilized in determining Poisson's ratio for frozen ground. Here, however, we encounter a serious difficulty; both modulus of elasticity and modulus of shear depend on the stress applied - the former on compressive stress, and the latter on shear stress.

The question arises as to whether Young's modulus would correspond to the modulus of shear even when under the same stress, but while one is under compression and the other is under shear.

If we assume that they do correspond, then, according to the preceding formula, we will also be able to determine Poisson's ratio μ , given the values E and G .

For frozen sand under a change of stress on the order of 2 kg./cm.², we have the following equations:

$$E = (0.5 + 2.1t) \times 10^4 \text{ kg./cm.}^2$$

$$G = (0.2 + 0.8t) \times 10^4 \text{ kg./cm.}^2$$

The above equations are valid at temperatures not lower than -5°C. Determining Poisson's ratio on the basis of the above data, at a temperature of -1°C., we arrive at:

$$E = 26,000 \text{ kg./cm.}^2 \text{ and } G = 10,000 \text{ kg./cm.}^2$$

Figure 12 is a diagram showing the relationship between G , modulus of shear, and temperature, varying from 0° to -4°C. This relationship, seen from Figure 12, may be considered linear within the limits of investigation; i.e., it may be supposed that at temperatures not lower than -5°C., the relationship between the modulus of shear and temperature may be expressed by the following formula:

$$G \approx \alpha_1 + \beta_1 t$$

where α_1 and β_1 are the constant coefficients, and t is the absolute value of the negative temperature in °C.

At a temperature not lower than -5°C. and with a shear stress τ of 2.3 kg./cm.², we will have:

$$G \approx (0.2 + 0.8t) \times 10^4 \text{ kg./cm.}^2$$

Thus, according to the known relationship, the formula for the modulus of shear is:

$$G = \frac{E}{2(1 + \mu)}$$

i.e., it is a function of Young's modulus, E , and Poisson's ratio, μ . Consequently,

then:

$$\mu = \frac{E}{2G} - 1 = \frac{26,000}{2 \times 10,000} - 1 = 0.30$$

Much in the same way we can determine that, at a temperature of $-5^{\circ}\text{C}.$, Poisson's ratio, μ , will be 0.31.

Table 8 shows the values of Poisson's ratio for frozen sand obtained by direct measurement by the above-described apparatus for simultaneous measurement of longitudinal and transverse deformation of prisms of frozen ground.

TABLE 8. POISSON'S RATIO FOR FROZEN SAND OBTAINED BY DIRECT MEASUREMENT.

Test No.	Moisture by weight, w, %	Temperature of sample, $^{\circ}\text{C}.$	Compressive stress, σ , kg./cm. ²	Poisson's ratio, μ
1	19.0	-0.8	6	0.13
2	19.0	-0.5	8	0.15
3	19.0	-0.3	2	0.41

If we take into consideration the relationship between the relative elastic deformation of frozen sand and external stress, we may consider the data obtained as not completely coinciding with but not contradictory to the above-cited values of Poisson's ratio. These values were obtained by means of calculation on the basis of the given values for E and G, whereas the values for Poisson's ratio in Table 8 are based on considerably larger stresses.

Elastic Deformation of Frozen Silt

The study of elastic deformation of frozen silt was conducted by the same method as that which was used for frozen sand; both the lever press and the oil press with constant application of load were used. Results of the investigation of deformation of prisms of frozen silt under compression are given in Table 9.

In addition to the relation between Young's modulus and compressive stress considered during the study of deformation of frozen sand, it is also necessary to point out the relation between Young's modulus for frozen silt and the rate of load increase. Thus, comparing tests 4, 5, and 6 with tests 7, 8, and 9 (Table 9), the conclusion is reached that Young's modulus increases with rate of load increase.

During the study of deformation of frozen silt, a supplementary series of tests (V) was conducted in which deformation of the prism of frozen silt was measured with the same sample at varying temperatures.

Influence of Negative Temperatures. Change in temperature of the sample greatly affects the elastic deformation of frozen silt. The relation between temperature and the modulus of elasticity of frozen silt is shown in Figure 13. As is seen from the data cited, this relationship may be graphically represented by a curve corresponding to an equation in the form of a polynomial of the third degree.

The first section of this curve, from -0.4° to $-4.0^{\circ}\text{C}.$, may be taken approximately as a straight line (with a sufficient degree of accuracy for all practical purposes). Therefore, in the equation of this curve:

$$E = \alpha + \beta t + \gamma t^2 + \delta t^3$$

the coefficients of t^2 and t^3 may be taken as equal to zero.

Determining the parameters of α and β for the first section of this curve (at a temperature not below $-4^{\circ}\text{C}.$), the following equation is obtained:

$$E_{0-4} = (0.4 + 1.4t) \times 10^4 \text{ kg./cm.}^2$$

where t is, as before, the exact negative temperature of the sample.

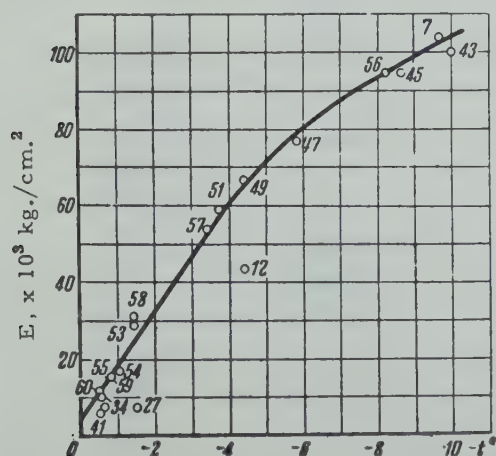


Fig. 13. Relationship between Young's modulus for frozen silt and temperature at $\sigma = 2 \text{ kg./cm.}^2$ ($E = \alpha + \beta t + \gamma t^2 + \delta t^3$ down to $-4^{\circ}\text{C}.$, $E \approx (0.4 + 1.4t) \times 10^4 \text{ kg./cm.}^2$)

Shear Modulus and Poisson's Ratio of Frozen Silt

The results of calculations of the elastic angle of torsion for samples of frozen silt at varying stresses and temperatures are given in Table 10.

On the basis of the data for the elastic angle of torsion and the geometrical dimensions of the samples of frozen ground, the modulus of shear (transverse elasticity) is given in the last column of Table 10.

TABLE 9. ELASTIC AND PLASTIC DEFORMATION OF FROZEN SILT UNDER COMPRESSION.

Test No.	Series	Moisture by weight, w, %	Temperature of sample, °C	Duration of load action, u	Stress, σ , kg./cm. ²	Young's modulus, E, kg./cm. ²	Stabilized rate of relative deformation, m, min. ⁻¹
1	2	3	4	5	6	7	8
1 ³	I	27.6	— 9.6	30" (5 times)	2	104 000	—
2		27.6	— 9.6		4	101 200	—
3		27.6	— 9.6		6	98 900	—
4 ⁸	I'	27.8	—10.0	Rate of load increase 2.4 kg/cm ² /min.	16	51 000	—
5		27.8	—10.0		32	44 000	—
6		27.8	—10.0		48	36 000	—
7		27.8	— 9.5	Rate of load increase 2.4 kg/cm ² /min.	10	48 000	—
8		27.8	— 9.5		20	35 000	—
9		27.8	— 9.5		30	27 000	—
10 ⁴		27.8	— 9.5		40	—	Complete fluidity
11	II	28.4	— 4.4	30" (5 times)	1	45 500	—
12		28.4	— 4.4		2	43 800	—
13		28.4	— 4.4		3	41 700	—
14 ⁵		28.4	— 4.4	Unloading	3	36 600	—
15 ⁶		28.4	— 4.5		3	41 500	—
16	II	27.3	— 4.0	30" (5 times)	2	48 800	—
17		27.3	— 4.0		4	36 300	—
18		27.3	— 4.0		6	32 400	—
19 ⁷		27.3	— 4.0	Unloading	6	33 900	—
20 ³		27.3	— 4.0	30" (5 times)	9	26 400	0.000000
21	II'	28.1	— 4.8		4	60 000	—
22		28.1	— 4.8	30" (3 times)	8	41 000	—
23		28.1	— 4.8		12	35 200	—
24		28.1	— 4.8		16	33 500	—
25		28.1	— 4.8		20	27 000	—
26	III	26.2	— 1.6	30" (3 times)	1	9 700	—
27		26.2	— 1.5		2	7 500	—
28 ⁹		26.2	— 1.5		3	6 800	0.000004
29		26.2	— 1.5		4	—	Flows constantly
30		25.8	— 1.1	60'	1	—	0.00000
31		25.8	— 1.0		2	—	0.000003
32		25.8	— 1.1		3	—	0.000014
33	IV	26.4	— 0.6	30" (5 times)	1	10 200	—
34		26.4	— 0.6		2	7 400	—
35		26.4	— 0.6		3	5 000	Flows constantly
36		27.1	— 0.6	60'	0.5	—	0.000000

1. Series I' and II' were done on the Amsler oil press.

2. Volumetric weight of frozen ground for tests 1 and 2, =1.75 g/cm³.3. Volumetric weight of frozen ground in tests 4 to 10, =1.56 g/cm³.

4. Sample lost stability.

5. Every 60'.

6. Every 120'.

7. Every 180'.

8. Observed for 970'.

9. Within margin of error of reading.

TABLE 9, continued.

Test No.	Series	Moisture by weight, w, %	Temperature of sample, °C	Duration of load action, u	Stress, σ , kg./cm. ²	Young's modulus, E, kg./cm. ²	Stabilized rate of relative deformation, m, min. ⁻¹
1	2	3	4	5	6	7	8*
37	IV	27.1	— 0.7	30" (5 times)	1	6 100	0.000003
38		27.1	— 0.4	60'	1.5	—	0.000086
39		26.5	— 0.5	30" (5 times)	1.0	8 700	—
40		26.5	— 0.5		1.5	7 600	—
41		26.5	— 0.5		2	6 200	0.000081
# 42 ¹		29.3	— 0.4		0.5	8 600	0.000004
43	V	30.0	— 10.0	30" (7 times)	2	100 000	—
44		30.0	— 10.0		4	91 000	—
45		30.0	— 8.6		2	95 200	—
46		30.0	— 8.6		4	85 100	—
47		30.0	— 5.8		2	76 800	—
48		30.0	— 5.8		4	62 500	—
49		30.0	— 4.4		2	66 600	—
50		30.0	— 4.4		4	51 300	—
51		30.0	— 3.7		2	58 800	—
52		30.0	— 2.6		4	41 700	—
53		30.0	— 1.4		2	28 600	—
54		30.0	— 1.0		2	16 900	—
55		30.0	— 0.5		2	10 200	—
56		29.6	— 8.2		2	95 200	—
57		29.6	— 3.4		2	54 000	—
58		29.6	— 1.4		2	30 800	—
59		29.6	— 0.8		2	15 400	—
60		29.6	— 0.5		2	10 500	—

TABLE 10. ELASTIC DEFORMATION AND MODULUS OF SHEAR OF FROZEN SILT UNDER TORSION.

Test No.	Moisture by weight, w, %	Temperature of sample, °C.	Load on sample, P, kg.	Greatest shear stress, τ , kg./cm. ²	Elastic increase of torsion angle, $\Delta\psi$, where $\ell = 5$ cm.	Modulus of shear, G, kg./cm. ²
1	25.2	— 11.3	8.0	9.1	0.00043	52,600
2	25.2	— 11.3	12.0	13.7	0.00106	42,000
3	27.0	— 3.4	1.0	1.1	0.00010	27,200
4	27.0	— 3.4	2.0	2.3	0.00035	23,300
5	27.0	— 3.4	5.0	5.7	0.00060	22,700
6	28.0	— 4.7	2.0	2.3	0.00016	32,300
7	28.0	— 4.6	6.0	6.8	0.00055	30,700
8	28.4	— 0.4	0.5	0.6	0.00025	5,400
9	28.4	— 0.4	1.0	1.1	0.00062	4,400
10	28.4	— 0.4	1.5	1.7	0.00101	4,000
11	26.8	— 0.4	0.5	0.6	0.00027	5,000
12	26.8	— 0.4	1.0	1.1	0.00056	4,900
13	26.8	— 0.4	1.5	1.7	0.00085	4,800

*m was determined at $t = -0.2^\circ\text{C}$.

According to Table 10, the modulus of shear decreased with increase of load, just as was the case for frozen sand. However, at a temperature of about -0.5°C ., the change in modulus of shear in relation to outside load is insignificant.

Figure 14 gives the relation between the shear modulus and temperature at P equals 1.5 to 2 kg. As can be seen from Figure 14, the relation between the shear modulus, G , and temperature down to about -4° or -5°C . can be taken as approximately linear.

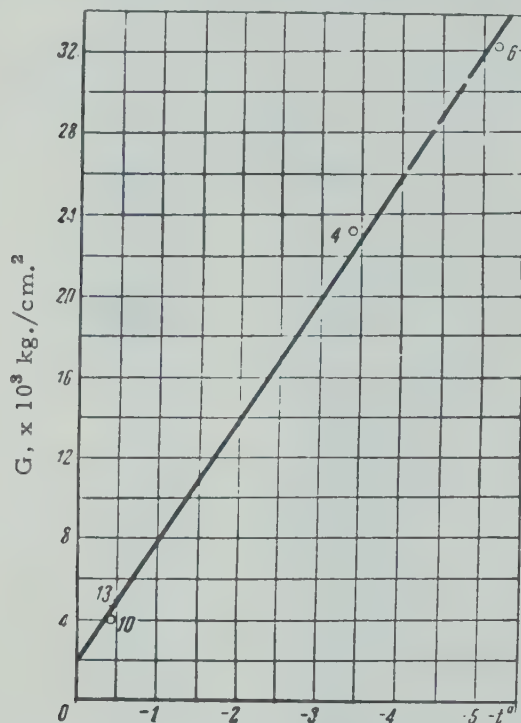


Fig. 14. Relationship between modulus of shear for frozen silt and temperature at $\tau = 2 \text{ kg./cm.}^2$ [$G = (0.16 + 0.6t) \times 10^4 \text{ kg./cm.}^2$]

For the case under consideration, $G \approx (0.16 + 0.6t) \times 10^4 \text{ kg./cm.}^2$

If we determine the value of Poisson's ratio on the basis of the data of Young's modulus and the modulus of shear, we obtain at the temperature $t = -1^{\circ}\text{C}$. the following equations:

$$E = (0.4 + 1.4t) \times 10^4 \text{ kg./cm.}^2 = 18,000 \text{ kg./cm.}^2$$

$$G = (0.16 + 0.6t) \times 10^4 \text{ kg./cm.}^2 = 7,600 \text{ kg./cm.}^2$$

$$\mu = \frac{E}{2G} - 1 = 0.18$$

At $t = -4^{\circ}\text{C}$., we get $\mu = 0.17$.

The values of Poisson's ratio for frozen silt, obtained by means of direct measurement of longitudinal and transverse deformation of a prism of frozen ground under compression, are cited in Table 11.

The data in Table 11 coincide rather well with values of Poisson's ratio (cited above) which were obtained by calculation on the basis of the values for Young's modulus and the shear modulus.

Elastic Deformation of Frozen Clay

Results of the tests for the study of deformation of prisms of frozen clay, the determination of Young's modulus, and the established rate of relative deformation under compression are given in Table 12.

Influence of Test Conditions. Analyzing the results of the determination of Young's modulus for frozen clay, it is

TABLE 11. RESULTS OF THE DIRECT DETERMINATION OF POISSON'S COEFFICIENT FOR FROZEN SILT

Test No.	Moisture by weight, w, %	Temperature of sample, $^{\circ}\text{C}$.	Compressive Stress, σ , kg./cm. 2	Poisson's ratio, μ
1	28.7	-4.0	6	0.13
2	25.3	-1.6	2	0.14
3	28.0	-0.8	2	0.16
4	28.0	-0.4	1.5	0.25

necessary to point out first, that it is small, and second, the relation between Young's modulus and the factors previously considered - load and temperature.

The tests for determining Young's modulus, conducted at the same rate of load increase as in tests 13 to 15 (see Table 12) - 2.5 kg./cm.^2 per minute - showed that in this case also it is a function of stress.

The control tests (Nos. 62 to 83) were aimed at checking the figures for Young's modulus by means of other types of apparatus. In general, these tests gave results which corresponded closely with the initial tests, namely, that frozen clay has a very low modulus of elasticity.

Several careful experiments were performed to determine Young's modulus for pure ice, frozen in the same containers and at the same temperatures as the samples of frozen clay. The value of Young's modulus for ice was obtained at a temperature of -1.5°C . and under a compressive stress of 2 kg./cm.^2 was found to lie between 23,000 and 25,600 kg./cm.^2 . Thus, the average value of E for ice was 24,500 kg./cm.^2 .

Under the same conditions (-1.5°C . and a compressive stress of 2 kg./cm.^2), the values of Young's modulus for frozen soils were as follows:

For frozen sand: $E = 36,500 \text{ kg./cm.}^2$

For frozen silt: $E = 25,000 \text{ kg./cm.}^2$

For frozen clay: $E = 8,300 \text{ kg./cm.}^2$

A comparison of these results, taking into consideration the fact that all the pores of the frozen ground were filled with water at the moment of freezing, leads to the conclusion that not nearly all the water in the frozen clay was transformed to ice, even though the ice crystals were equally distributed throughout the entire sample of clay according to direct observations.

Influence of Temperature. Let us consider the relation between Young's modulus for frozen clay and temperature. In this case, just as before, it is necessary to take into consideration the effect of varying loads on the value of Young's modulus for samples at equal temperatures.

Figure 15 is a graph of the relation of temperature to the modulus of elasticity of frozen clay under a load of 2 kg./cm.^2 . Figure 15 shows this relation to be linear at temperatures between -0.4° and -5°C .

A determination of the parameters of the line expressing the relation between temperature and Young's modulus for frozen clay gives us the following:

$$E \approx (0.5 + 0.23t) \times 10^4 \text{ kg./cm.}^2$$

If, however, a similar graph is drawn on the basis of tests where the compressive stress is 1 kg./cm.^2 , then, determining the parameters of the straight line within the limits of -0.5° to -5°C ., we will get the following:

$$E \approx (0.7 + 0.23t) \times 10^4 \text{ kg./cm.}^2$$

Comparing the second equation with the first, we can see that they differ only in the initial coefficients. The angle coefficient (of t) remains unchanged. Here it may be assumed that the load and the degree of freezing of the clay primarily affect the initial coefficient of the line $E = f(t)$. The tests on clay, previously cited,¹ where grain-size composition was close to that of those under consideration but where the samples were subjected to freezing for a period of not less than three days, produced the following relationship between Young's modulus and temperature:

$$E = (1.1 + 0.24t) \times 10^4 \text{ kg./cm.}^2$$

Fig. 15. Relationship between Young's modulus for frozen clay and temperature at $\sigma = 2 \text{ kg./cm.}^2$
($E \approx (0.5 + 0.23t) \times 10^4 \text{ kg./cm.}^2$)

Comparison of this equation with the preceding one shows that the difference lies primarily in the initial coefficient. Therefore, the longer period of freezing of the clay resulted in an increase in Young's modulus and produced practically no change in the curvature of the line which expresses the relationship between Young's modulus and temperature.

Shear Modulus and Poisson's Ratio for Frozen Clay

The samples of frozen clay used in the study of elastic properties under shear only were of less than half the volume of those used for the study of elastic properties under perpendicular (compressive) stress. In addition, in the torsion tests, the central part of the samples had a thickness of only 4 cm., a fact which apparently was conducive to better freezing of the water in the pores of the clay.

The results of tests for studying elastic properties of frozen clay under torsion are given in Table 13.

1. Laboratornye issledovaniia mekhanicheskikh svoistv merzlykh gruntov (Laboratory Researches into the Mechanical Properties of Frozen Ground), Academy of Sciences, 1936.

TABLE 12. ELASTIC AND PLASTIC DEFORMATION OF FROZEN CLAY UNDER COMPRESSION

Test No.	Series	Moisture by weight, w, %	Temperature of sample, °C.	Duration of load action, u	Stress, σ , kg./cm. ²	Young's modulus, E, kg./cm. ²	Stabilized rate of relative deformation, m, min. ⁻¹
1	2	3	4	5	6	7	8
1	I	52.1	-8.4	30" (5 times)	1	27 800	—
2		52.1	-8.4		2	25 600	—
3		52.1	-8.4		4	23 900	—
4		52.1	-8.4		6	22 500	—
5 ¹		52.1	-8.4	Unloading	6	22 800	—
6		59.2	-8.2	30" (5 times)	4	23 300	—
7		59.2	-8.2		6	21 500	—
8		59.2	-8.2		8	20 100	0.000003
9 ¹		59.2	-8.2	Unloading	8	18 000	—
10 ²	I'	53.0	-9.2	15—30"	5	20 000	—
11		53.0	-9.2		10	17 200	—
12		53.0	-9.2		15	15 800	—
13 ³		53.0	-10.4	120" (5 times)	5	20 800	—
14		53.0	-10.4	240" (5 times)	10	18 500	—
15		53.0	-10.4	360" (5 times)	15	16 000	—
16		53.0	-10.4	480" (5 times)	20	Complete fluidity	—
17	II	46.6	-5.1	30" (5 times)	1	17 200	—
18	II	46.6	-5.1	Unloading	1.5	14 300	—
19 ⁴		46.6	-4.6		1.5	14 300	—
20		45.7	-5.2	30" (5 times)	1	19 200	—
21		45.7	-5.2		2	17 100	—
22		45.7	-4.5		3	15 800	—
23 ⁵		46.7	-4.5	90'	3	—	0.000002
24		46.7	-4.7	120'	4	—	0.000003
25		46.7	-5.0	1260'	5	—	0.000004
26		46.7	-5.0	60'	6	—	0.000007
27		46.7	-5.0	60'	7	—	0.000019
28		46.7	-5.0	60'	8	—	0.000026
29		46.7	-5.0	90'	9	12 200	0.000091
30	III	43.1	-2.7	30" (5 times)	2	9 100	—
31		43.1	-2.7		3	8 200	—
32		43.1	-2.7		3	—	0.000034
33 ⁶		58.1	(-2.1)	30" (5 times)	1	12 800	—
34		58.1	(-1.9)		2	10 900	Progressive flow
35		49.6	-1.7	30" (5 times)	0.5	8 900	—
36 ⁷		49.6	-1.2		1	8 300	0.000004
37 ³		49.6	-1.2		2	6 800	0.000024
38 ⁹		53.7	-1.4	30" (5 times)	0.5	9 600	0.000004
39 ⁷		53.7	-1.4		1	9 400	0.000008
40 ⁷		53.7	-1.5		2	7 900	—

1. Every 120'.

2. Series I' were conducted on the Amsler oil press.

3. Tests 13 to 15 were done at a constant load increase rate, $v=2.5$ kg./cm.²/min.

4. Every 180'.

5. In tests 23 to 29, deformations were measured every 5'.

6. The value of t in parentheses corresponds to the temperature of the chamber.

7. At $u=60'$.8. At $u=60'$ and $t=-1.5^\circ\text{C}$.9. At $u=100'$.

TABLE 12. continued

Test No.	Series	Moisture by weight, w, %	Temperature of sample, °C.	Duration of load action, u	Stress, σ , kg./cm. ²	Young's modulus, E, kg./cm. ²	Stabilized rate of relative deformation, m, min. ⁻¹
1	2	3	4	5	6	7	8
41 ¹	III	56.1	-1.9	60'	0.5	—	0.000000
42		56.1	-1.9		1	—	0.000002
43		56.1	-2.0		1.5	—	0.000008
44		56.1	-1.9		2	—	0.000020
45		56.1	-1.9		2.5	—	>0.000130
							Progressive flow
46	IV	48.0	-0.7	30" (5 times)	0.5	10 000	—
47		48.0	-0.7		1	9 500	—
48		48.0	-0.5		2	7 000	—
49 ¹		48.0	-0.4		3	6 700	Flow
50		53.0	-0.4		0.5	9 800	—
51		53.0	-0.6	60'	1	8 600	—
52		53.0	-0.6		2	8 000	—
53 ²		53.0	-0.3		3	—	>0.000173
54	V	~54.0	-10.1	30" (5 times)	2	24 900	—
55		~54.0	-7.5		2	21 900	—
56		~54.0	-6.2		2	16 300	—
57		~54.0	-4.7		2	13 700	—
58		~54.0	-2.0		2	10 500	—
59		~54.0	-8.0		2	22 200	—
60		~54.0	-5.0		2	16 700	—
61		~54.0	-0.4		2	6 400	—
62	Control	53.7	-4.9	30" (5 times)	2	16 300	—
63		53.7	-1.4		2	7 800	—
64		53.7	-1.2		2	6 600	—
65	№ 2 ³	54.7	-1.6	30" (5 times)	0.5	7 300	—
66		54.7	-1.7		1	7 800	—
67		54.7	-1.7		1.5	8 400	—
68		54.7	-1.8		2	8 400	—
69		54.7	-1.6		2.5	7 400	—
70		55.1	-1.6		0.5	8 100	—
71		55.1	-1.6		1	8 900	—
72		55.1	-1.5		1.5	8 400	—
73		55.1	-1.5		2	7 300	—
74		55.1	-1.5		2.5	6 800	—
75		55.8	-5.1		0.5	19 300	—
76		55.8	-5.0		1	17 900	—
77		55.8	-5.2		1.5	17 100	—
78		55.8	-5.0		2	15 600	—
79		55.8	-5.1		2.5	14 000	—
80		64.2	-5.1		0.5	19 300	—
81		64.2	-5.0		1	17 600	—
82		64.2	-4.9		1.5	17 000	—
83		64.2	-5.1		2	15 000	—
84 ⁴		Ice	-1.5		2	24 500	—

1. In tests 41 to 45, the deformations were measured every 5'. The average value of two measurements is given.

2. Progressive flow.

3. The measuring of deformation in control series No. 2 was done with Martens mirror apparatus.

4. Pure ice; tests were repeated three times.

TABLE 13. ELASTIC DEFORMATION AND MODULUS OF SHEAR OF FROZEN CLAY UNDER TORSION

No.	Moisture by weight, w, %	Temperature of sample, °C.	Load on sample, P, kg.	Greatest shear stress, τ , kg/cm. ²	Elastic increase of torsion angle, $\Delta\psi$, where $l=5$ cm.	Modulus of shear, G, kg./cm. ²
1	50.7	-10.4	2.0	2.3	0.00047	11,600
2	50.7	-10.4	4.0	4.6	0.00132	8,200
3	50.7	-10.4	6.0	6.8	0.00234	7,000
4	55.8	-10.6	1.0	1.1	0.00025	11,500
5	55.8	-10.6	4.0	4.6	0.00150	7,700
6	55.8	-10.6	6.0	6.8	0.00269	5,800
7	54.8	-4.7	2.0	2.3	0.00064	9,000
8	54.8	-4.7	3.0	3.4	0.00122	7,100
9	57.7	-1.0	1.0	1.1	0.00036	7,500
10	57.7	-1.2	2.0	2.3	0.00071	7,700
11	57.7	-1.2	3.0	3.4	0.00122	6,700
12	53.7	-1.0	0.5	0.6	0.00010	13,600
13	53.7	-1.0	2.0	2.3	0.00074	7,300

From a study of the data in Table 13, it may be seen that the shear modulus of frozen clay (as in the case of Young's modulus) decreases with increased load and increases with decreased temperature almost on a linear basis (within the limits of our study).

Due to the difference in the degree of freezing of the samples used in the study of Young's modulus and shear modulus, we feel that comparison of shear modulus with Young's modulus and the calculation of Poisson's ratio of frozen clay on this basis is not sufficiently justified.

The direct determination of Poisson's ratio during testing the prisms under compression gave the following results.

A sample with a moisture content of 49.5% at a temperature of -5°C. and under a compressive stress (σ) of 12 kg./cm.² showed $\mu = 0.26$. A sample with a moisture content of 53.4% at a temperature of -1.7°C. and under a stress of $\sigma = 4$ kg./cm.² showed $\mu = 0.35$.

Calculation of Poisson's ratio for frozen clay based on several compressive strength tests on an area of 25 cm.² (utilizing the Boussinesq-Schleicher formula) at -1.5°C. showed $\mu = 0.36$ to 0.48.

Thus, the tests to determine Poisson's ratio for frozen ground show how difficult it is to establish the value of this coefficient, since it is subject to the influence of a series of factors.

V. PLASTIC DEFORMATION OF FROZEN GROUND AND ITS TEMPERATURE BORDER WITH ELASTIC DEFORMATION

Plastic deformation of three typical varieties of frozen ground was studied simultaneously with elastic deformation of the same types of ground. Plastic deformation is residual deformation which takes place without the destruction of the tested matter.

In the tests which have been described, the constancy of the rate of deformation during a prolonged period of time (not less than one-half to one hour for prisms measuring 7 x 7 x 15 cm.) was taken as the criterion for plastic deformation.

Table 14 gives the average values for the stabilized rate of relative deformation (per unit length) for frozen sand, all pores of which are filled with ice ($w = 17$ to 22%).

Figure 16 gives the curve for the relationship between the rate of relative deformation of frozen sand and compressive stress (the tests are numbered as in Table 6).

TABLE 14. STABILIZED RATE OF RELATIVE DEFORMATION
FOR FROZEN SAND UNDER COMPRESSION

No.*	Compressive stress, σ , kg./cm. ²	Temperature of sample, °C.	Stablized rate of deformation, m, min. ⁻¹	Test numbers used as basis for given values (see Table 6)
1	1	-0.6	0.000010	54,56
2	2	-0.6	0.000045	59,52
3	3	-0.6	0.000094	55,57
4	2	-1.6	0.000005	35,36
5	3	-1.9	0.000021	29,33,37
6	4	-1.5	0.000043	30,34,38
7	1	-1.5	0.000000	68
8	1.5	-1.8	0.000002	69
9	2.0	-1.8	0.000008	70
10	2.5	-1.5	0.000013	71
11	3.5	-1.5	0.000027	72
12	5.0	-1.6	Complete loss of stability	73

*Tests 7 to 12 give the average of two tests.

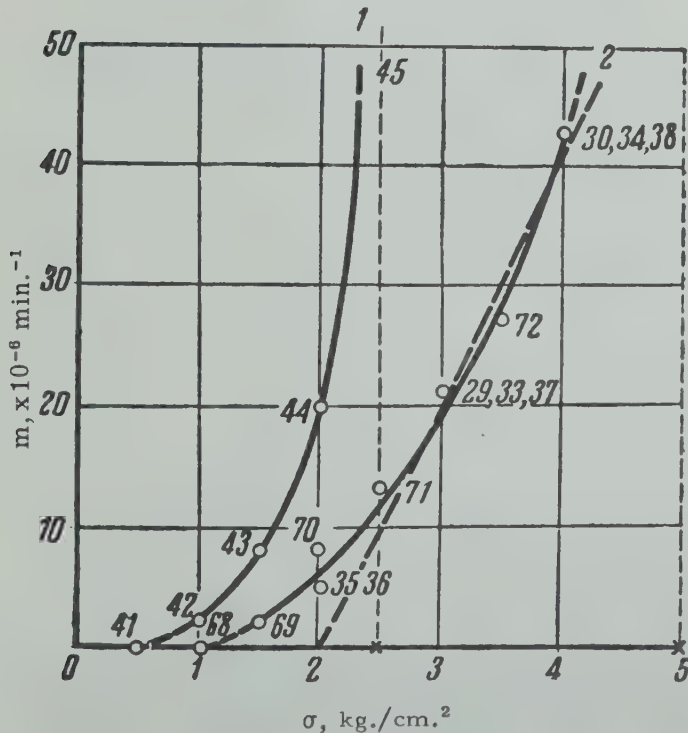


Fig. 16. Relationship between the rate of relative deformation of frozen ground and compressive stress. (Curve 1, frozen clay at average temperature of -1.9°C ., according to Table 12; curve 2, frozen sand at average temperature of -1.6°C ., according to Table 6.)

If a straight line be drawn between the last three points and continued to the intersection of the axis, σ , as is usually done for estimating critical stress for plastic flow, the following value will be obtained:

$$\sigma_{cr} \approx 2 \text{ kg./cm.}^2 \text{ at } t = -1.6^\circ\text{C}.$$

The same value can be established by comparing the data of Table 14.

When the compressive stress is 2 kg./cm.^2 and the temperature is -1.6°C ., frozen sand, free to expand, develops a deformation which does not diminish in the course of time, i.e., the sand flows constantly.

Under a smaller degree of stress and at the same temperature, the frozen sand will undergo only elastic deformation for the most part.

Therefore, with a load of 2 kg./cm.^2 , a temperature of -1.5° or -1.6°C . will constitute the border between a state of primarily elastic deformation (at lower temperature) and the state of primarily plastic deformation (at higher temperature) for frozen sand.

The data of Table 6 show that with a load of 6 kg./cm.^2 , the temperature border of plastic deformation for frozen sand will be -5°C . At a pressure of 1 kg./cm.^2 , this temperature border of plastic deformation approaches zero (under the condition of free expansion and a temperature not lower than -0.6°C .)

If we compare the limits of temperature within which plastic deformation occurs with

the strength of a cube of the ground, as given in Figure 7:

$$\sigma_b = 88 \text{ kg./cm.}^2 \text{ at } t = -5^\circ\text{C.}$$

$$\sigma_b = 36 \text{ kg./cm.}^2 \text{ at } t = -1.5^\circ\text{C.}$$

$$\sigma_b = 18 \text{ kg./cm.}^2 \text{ at } t = -0.6^\circ\text{C.}$$

then we can see that the dangerous stress causing plastic deformation will be approximately 15 to 18 times smaller than the destructive stress which was obtained during the testing of the cubes under compression at the standard rate of load increase. The critical stress corresponds to the initial appearance of the stabilized plastic flow. If we continue to increase the load on the sample, then, following the elastic and plastic stages of the deformation, the third state, the stage of progressive flow, will take place. The third stage is characterized by the ever-increasing intensity of deformation, resulting in excessive change in form of the samples and a complete loss of stability.

The state of progressive flow for frozen sand occurs, according to data in Tables 6 and 14 and Figure 16, at a temperature of -1.6°C and at a compressive stress $\sigma = 5 \text{ kg./cm.}^2$, i.e., 2 1/2 times greater than is needed for the start of the stabilized plastic flow, occurring for frozen sand at $\sigma = 2 \text{ kg./cm.}^2$.

The progressive flow stage, according to Table 9, occurs for frozen silt at a temperature of -1.5°C . and a moisture content (w) of 25.8% under a load of 4 kg./cm.^2 . However, at a temperature of -0.6°C . with $w = 26.4\%$, this stage occurs at a stress not greater than 3 kg./cm.^2 .

The stabilized rate of relative deformation for frozen silt is approximately twice that of frozen sand.

Thus, according to Table 9, at a temperature of -0.5°C ., a moisture content of 26.5%, and a stress of 2 kg./cm.^2 , the stabilized rate of relative deformation of frozen silt is $m = 0.000081 \text{ min.}^{-1}$. However, for frozen sand, approximately under the same conditions (No. 2 in Table 14) and load, $m = 0.000045 \text{ min.}^{-1}$.

The last figure shows that the coefficient of viscosity of frozen silt is approximately half that of frozen sand.

Plastic deformation of frozen clay was studied simultaneously with elastic deformation. Results of the determinations of the stabilized rate of deformation are given in Table 12 and are compared in Table 15.

TABLE 15. STABILIZED RATE OF RELATIVE DEFORMATION
FOR FROZEN CLAY UNDER COMPRESSION

No.	Temperature, $^\circ\text{C}$.		Stabilized rate of relative deformation, $m, \text{ min.}^{-1} \times 10^6$										
	From	To	Stress, kg./cm.^2										
			1	1.5	2	2.5	3	4	5	6	7	8	9
1	-4.5	-5	--	--	--	--	2	3	4	7	19	26	91
2	-2.7		--	--	--	--	34	--					
3	-1.2	-1.4	6	--	24	--	--	--	--	--	--	--	--
4	-1.9	-2.0	2	8	20	>130	--	--	--	--	--	--	--
5	-0.3		--	--	--	--	>173	--	--	--	--	--	--

The relationship between rate of relative deformation of frozen clay and compressive stress is plotted in Figure 16, on the basis of data in test 4 of Table 15. As can readily be seen, this relationship is expressed by a curve.

On the basis of data in Tables 12 and 15 and Figure 16, 1 kg./cm.^2 may be taken as the critical stress for frozen clay to start plastic flow at -1.9°C .

The progressive flow stage at the same temperature (-1.9°C .) occurs when $\sigma = 2.5 \text{ kg./cm.}^2$.

Consequently, at a temperature varying between -1.5° and -1.9°C ., the progressive flow stage occurs in frozen sand at $\sigma = 5 \text{ kg./cm.}^2$; in frozen silt at $\sigma = 4 \text{ kg./cm.}^2$; and in frozen clay at $\sigma = 2.5 \text{ kg./cm.}^2$.

The results of comparing the values of the established rate of relative deformation of frozen sand and frozen clay at temperatures varying between -1.5° and -2.0°C . are given in Table 16.

These data show that the coefficient of viscosity of frozen clay is no more than one-third or one-fourth that of frozen sand.

Therefore, given an equal load greater than the critical load, plastic deformation will be greatest for frozen clay and smallest for frozen sand. The plastic deformation of frozen silt during the same period of time will occupy an intermediate position between that of clay and sand.

TABLE 16. COMPARISON OF STABILIZED RATE OF RELATIVE DEFORMATION OF FROZEN SAND AND CLAY UNDER COMPRESSION BETWEEN -1.5° AND -2.0°C .

Stress, σ , kg./cm. ²	Stabilized rate of relative deformation, m, min. ⁻¹	
	Sand	Clay
1.5	0.000002	0.000008
2.0	0.000008	0.000020
2.5	0.000013	>0.000130

Plastic deformation will increase constantly, and the total deformation of frozen ground will be directly proportional to the time elapsed from the application of load.

As far as the elastic deformation of the types of frozen ground studied is concerned, it may be considered, for all practical purposes, to be independent of elapsed time and to occur both in the zone of elastic deformation and in the entire zone of plastic deformation as well. The following examples will confirm these conclusions.

According to Table 6, Young's modulus for frozen sand, E , is 190,000 kg./cm.² at -10.6°C . and a stress of $\sigma = 10$ kg./cm.² After being under load for an hour and subsequent unloading, Young's modulus at -10.8°C . was of the same order, i.e., $E = 190,000$ kg./cm.² After 2,760 minutes at -9.7°C . and after

removal of load, Young's modulus was $E = 161,300$ kg./cm.²

According to Table 9, Young's modulus for silt at -4.4°C . and a stress of $\sigma = 3$ kg./cm.² was $E = 41,700$ kg./cm.²; after 120 minutes at -4.5°C . and after removal of load, it was 41,500 kg./cm.²

Much in the same way as in the case of frozen clay, according to Table 12 (tests 4, 5, 8, 9, 18, and 19), the elastic properties and the value of Young's modulus are retained in the plastic phase of deformation.

VI. PRACTICAL DEDUCTIONS BASED ON RESULTS OF STUDIES

On the basis of these tests on the elastic and plastic properties of three typical varieties of frozen ground, we arrive at the following conclusions.

1. The tests established that Young's modulus for frozen ground decreases with increased load, given the same temperatures. This circumstance must be taken into consideration during the planning of structures subject to dynamic load.

2. The change in Young's modulus for frozen ground with temperature is curvilinear in pattern. However, at temperatures between -0.3° and -5°C ., the relationship of Young's modulus and shear modulus to temperature may be considered linear for all practical purposes.

3. Three phases of deformation occurring at definite intervals may be established when a load is applied to frozen ground: elastic deformation, plastic deformation, and progressive flow.

Purely elastic deformation takes place only under negligible stress in the temperature range under consideration. Thus, for the samples of frozen ground under investigation, elastic deformation under the condition of unhampered expansion of the ground occurs at about -1.5°C . at a stress of up to 2 kg./cm.² for sand and up to 1 kg./cm.² for clay. Loads at least two to three times larger will result in no more than elastic deformation where lateral expansion is restricted (for example, in the case of pressure on an area within a large expanse of frozen ground).

The elastic properties of frozen ground are preserved throughout the whole plastic deformation stage and even throughout the major part of the progressive load stage.

The plastic deformation stage of frozen ground occurs only at a given load for each level of temperature and is characterized by a constant rate of deformation. Given free expansion and a temperature of about -1.5°C ., plastic deformation of frozen sand was observed when the load attained 2 kg./cm.² and more, while frozen clay showed plastic deformation at loads over 1 kg./cm.²

However, under conditions of restricted expansion (such as pressing a stamp into the mass), the critical points for plastic deformation will be several times greater, as has been demonstrated by experiments conducted by us as well as by theoretical considerations.

In terms of required critical load, the progressive flow state follows that of the plastic deformation stage. It is characterized by an increase in rate of deformation which causes excessive changes in the form of the sample under test, often resulting in the loss of stability or destruction of the structure. Under conditions of free expansion of samples of frozen ground and a range of temperatures from -1.5° to -1.9°C ., progressive flow was observed at a stress of 5 kg./cm.² for frozen sand, 4 kg./cm.² for frozen silt, and 2.5 kg./cm.² for frozen clay.

4. The viscosity of frozen ground is insignificant, and the coefficient of viscosity has a value varying from 10^{12} to 10^{13} grams second/cm.², i.e., approximately the same as the coefficient of viscosity for pure ice.

This shows that plastic flow similar to glacial flow may arise in a mass of frozen ground under the action of its own weight. This flow may cause long-range deformation of the whole stratum of frozen ground which will undoubtedly have an adverse effect upon structural strength and stability. This applies particularly to structures erected on slopes where there is the greatest possibility of flow of frozen ground.

5. The tests demonstrated that all the types of frozen ground subjected to study, regardless of grain-size composition, have identical compressive and shear strengths at temperatures between 0° and -0.5°C., a fact of considerable practical significance.

An English summary of 16 lines is given.

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